Clicker Questions

Modern Physics
Chapter 5: "The Schrödinger Equation"
Cambridge University Press
felderbooks.com

by Gary Felder and Kenny Felder

Instructions

- These questions are offered in two formats: a deck of PowerPoint slides, and a PDF file. The two files contain identical contents. There are similar files for each of the 14 chapters in the book, for a total of 28 files.
- Each question is marked as a "Quick Check" or "ConcepTest."
  - Quick Checks are questions that most students should be able to answer correctly if they have done the reading or followed the lecture. You can use them to make sure students are where you think they are before you move on.
  - ConcepTests (a term coined by Eric Mazur) are intended to stimulate debate, so you don't want to prep the class too explicitly before asking them. Ideally you want between 30% and 80% of the class to answer correctly.
- Either way, if a strong majority answers correctly, you can briefly discuss the answer and move on. If many students do not answer correctly, consider having them talk briefly in pairs or small groups and then vote again. You may be surprised at how much a minute of unguided discussion improves the hit rate.
- Each question is shown on two slides: the first shows only the question, and the second adds the correct answer.
- Some of these questions are also included in the book under "Conceptual Questions and ConcepTests," but this file contains additional questions that are not in the book.
- Some of the pages contain multiple questions with the same set of options. These questions are numbered as separate questions on the page.
- Some questions can have multiple answers. (These are all clearly marked with the phrase "Choose all that apply.") If you are using a clicker system that doesn't allow multiple responses, you can ask each part separately as a yes-or-no question.

5.1 Force and Potential Energy

Describing the “potential energy field” around a particle is equivalent to describing. . . (Choose one.)

A. The way that particle is moving.

B. The forces acting on that particle.

C. Both of these.

D. Neither of these.

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C. Both of these.

D. Neither of these.

Solution: B

The relationship between force and potential energy can be briefly summarized as. . . (Choose one.)

A. High potential energy means strong force.

B. Low potential energy means strong force.

C. The force tends to push toward regions of high potential energy.

D. The force tends to push toward regions of low potential energy.

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Solution: D

An object is moving in a potential field shaped like $U(x) = \sin x$, starting at position $x(0) = 3\pi/2$. Which of the following will *definitely not happen*, no matter what the object's initial velocity? (Choose one.)

A. The object will come to rest at $x = -\pi/2$.

B. The object will remain at rest at $x = 3\pi/2$.

C. The object will oscillate forever.

D. The object will move leftward forever.

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B. The object will remain at rest at $x = 3\pi/2$.

C. The object will oscillate forever.

D. The object will move leftward forever.

Solution: A. The object is starting at a minimum of the potential energy. If the object starts with zero velocity it will remain at $x = 3\pi/2$. If it starts with nonzero velocity, but not enough to get out of the valley it's in, then it will oscillate forever. If it starts moving leftward fast enough to get over the maximum, it will keep moving leftward forever. But to move at all it will need an initial kinetic energy, and it will reach $x = -\pi/2$ with that same kinetic energy, so it will keep moving.

If we tell you that “At $x = 10$ this object’s potential energy is negative,” what can you conclude about the behavior of the object? (Choose one.)

A. The object will tend to move toward $x = 10$.

B. The object will tend to move away from $x = 10$.

C. The object cannot physically be at $x = 10$.

D. You won’t tell me that, because a negative potential energy is impossible.

E. A negative potential energy is possible, but it doesn’t physically tell you anything.

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D. You won’t tell me that, because a negative potential energy is impossible.

E. A negative potential energy is possible, but it doesn’t physically tell you anything.

Solution: E

5.2 Energy Eigenstates and the Time-Independent Schrödinger Equation

Which of the following are properties of an energy eigenstate? (Choose all that apply.)

A. It gives you a definite answer for a particle's position.

B. It gives you a definite answer for a particle's momentum.

C. It gives you a definite answer for a particle's energy.

D. It doesn't change over time (unless something external to the system causes it to change).

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B. It gives you a definite answer for a particle's momentum.

C. It gives you a definite answer for a particle's energy.

D. It doesn't change over time (unless something external to the system causes it to change).

Solution: C, D

When you plug a potential energy field $U(x)$ into Schrödinger's equation and then solve the resulting differential equation, you find... (Choose one.)

A. The wavefunction of a particle in that potential energy field.

B. The most probable wavefunction of a particle in that potential energy field.

C. The ground state (lowest energy eigenstate) of a particle in that potential energy field.

D. All the energy eigenstates of a particle in that potential energy field.

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C. The ground state (lowest energy eigenstate) of a particle in that potential energy field.

D. All the energy eigenstates of a particle in that potential energy field.

Solution: D

The axioms of quantum mechanics stipulate that… (Choose all that apply.)

A. The wavefunction $\psi(x)$ must be everywhere continuous.

B. The wavefunction $\psi(x)$ must be everywhere differentiable.

C. The wavefunction $\psi(x)$ must be infinitely differentiable. (This means that its second derivative, its third derivative, and so on, are all defined everywhere.)

D. The wavefunction $\psi(x)$ must be normalized.

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D. The wavefunction $\psi(x)$ must be normalized.

Solution: A, B, and D

Is it possible for a particle to be in a state in which any energy from 0 to ∞ is just as likely as any other energy?

Is it possible for a particle to be in a state in which any energy from 0 to ∞ is just as likely as any other energy?

Solution: No, that would not be normalizable.

Can an energy eigenvalue be negative? Why or why not?

Can an energy eigenvalue be negative? Why or why not?

Solution: Yes, because potential energy (and thus total energy) can be negative.

You have been given the potential energy function $U(x)$ around an object, and you have found a function $\psi(x)$ that satisfies Schrödinger equation for that $U(x)$. You have also determined that $\int_{-\infty}^{\infty} |\psi(x)|^2 dx$ is 9. What do you conclude?

A. You can just divide your $\psi(x)$ function by 3. The resulting function will still solve Schrödinger's equation, and it will be properly normalized, so it represents the wavefunction.

B. You need to start over with Schrödinger's equation and find a completely different wavefunction; this one is not valid.

C. Your $\psi(x)$ function is fine the way it is.

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B. You need to start over with Schrödinger's equation and find a completely different wavefunction; this one is not valid.

C. Your $\psi(x)$ function is fine the way it is.

Solution: A

Does the ground state of a system have a nonzero energy uncertainty associated with it? (Choose one.)

A. Yes, all states have some energy uncertainty.

B. No, the ground state always has a definite value of energy.

C. Some systems have a ground state with definite energy and others have a ground state with some energy uncertainty.

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A. Yes, all states have some energy uncertainty.

B. No, the ground state always has a definite value of energy.

C. Some systems have a ground state with definite energy and others have a ground state with some energy uncertainty.

Solution: B.

You might very reasonably give the same answer for the first excited state, and the second excited state, and so on. The definition of an energy eigenstate is its lack of energy uncertainty. A particle has an uncertain energy when it is in a superposition of different energy eigenstates.

But as a more advanced note, actual measured eigenstates do have a small uncertainty (or "width")—*except* the ground state. That one truly is definite, as you can conclude from the time-energy uncertainty principle: a ground state will never decay (since it has nowhere to decay to), so it must have zero energy uncertainty.

5.3 The Infinite Square Well

When we solved for the energy eigenfunctions of a particle in a square well, the resulting energy was quantized because. . . (Choose one.)

A. The solution to Schrödinger's equation produced quantized values of E.

B. The boundary conditions produced quantized values of E.

C. We imposed quantization as a requirement after the algebra was done.

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A. The solution to Schrödinger's equation produced quantized values of E.

B. The boundary conditions produced quantized values of E.

C. We imposed quantization as a requirement after the algebra was done.

Solution: B

Equation 5.8 on p. 230 represents what about a particle in an infinite square well? (Choose all that apply.)

A. All the possible wavefunctions the particle could have

B. The wavefunctions that have definite values of kinetic energy

C. The wavefunctions that have definite values of total energy

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A. All the possible wavefunctions the particle could have

B. The wavefunctions that have definite values of kinetic energy

C. The wavefunctions that have definite values of total energy

Solution: B, C

We found that the energy of our particle in a box can be $\hbar^2\pi^2 n^2/(2mL^2)$. In this equation, the quantum number n can be. . . (Choose one.)

A. Any real or complex number

B. Any real number

C. Any positive integer

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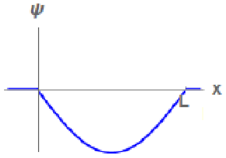
A. Any real or complex number

B. Any real number

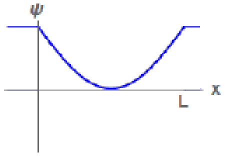
C. Any positive integer

Solution: C

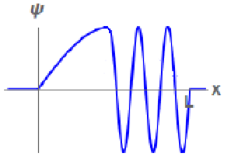
Which of the following are possible wavefunctions in an infinite square well? (Choose all that apply.)



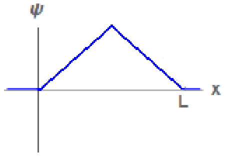
A



B

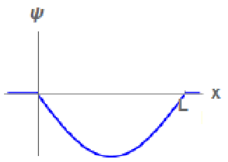


C

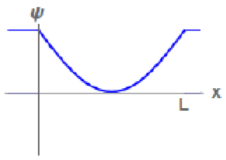


D

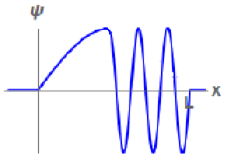
Which of the following are possible wavefunctions in an infinite square well? (Choose all that apply.)



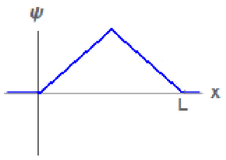
A



B



C



D

Solution: A and C. Continuity at the boundaries requires that $\psi(x) = 0$ at the two edges of the well. Differentiability inside the well rules out D. Because of the infinite potential energy jump, the wavefunction does not need to be differentiable at the boundaries.

The energy eigenstates we found for the infinite square well involved only sines, no cosines. Suppose that instead of going from 0 to L the well went from $-L$ to L. Would the energy eigenstates involve. . . (Choose one.)

A. only sines

B. only cosines

C. both sines and cosines

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A. only sines

B. only cosines

C. both sines and cosines

Solution: C. The restriction to sines came because they needed to equal zero at both 0 and L, and only a sine equals zero at $x = 0$. Both sines and cosines can equal zero at $-L$ and L. For example, $\sin(\pi x/L)$ would work, but so would $\cos(\pi x/(2L))$, so the energy eigenstates will involve both sines and cosines.

You have a thousand identical boxes, each of which contains a particle in an infinite square well. The first five hundred boxes are labeled A and the second five hundred B. At $t = 0$ you open all of the A boxes and measure the particle's positions. At $t = 1$ (one hour later) you do the same thing to all the B boxes. In each of the cases below, will you find the same distribution of positions in the A group and the B group?

1. Initially all the particles were in the ground state.
2. Initially all the particles were in the state $1/\sqrt{2}(\psi_1 + \psi_2)$ where ψ_1 is the ground state and ψ_2 is the first excited state.

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1. Initially all the particles were in the ground state.

   Solution: Yes. An eigenstate of energy is a stationary state, meaning that the probabilities of any measurable quantity (such as position) will not change over time if nothing is disturbed.

2. Initially all the particles were in the state $1/\sqrt{2}(\psi_1 + \psi_2)$ where ψ_1 is the ground state and ψ_2 is the first excited state.

   Solution: No, a superposition of eigenstates is not in general stationary, so the position probabilities will change over time.

We'll use ψ_n to represent the n^{th} energy eigenstate of a particle in an infinite square well. For example, $\psi_2 = \sqrt{2/L}\sin(2\pi x/L)$ inside the well. Which of the following is a possible wavefunction for the particle? (Choose all that apply.)

A. ψ_1

B. ψ_2

C. $\psi_1 + \psi_2$

D. $(1/\sqrt{2})(\psi_1 + \psi_2)$

E. $(1/\sqrt{2})(\psi_1 - \psi_2)$

F. $Ax(L - x)$ (Choose this option if there is any value of A for which this is possible.)

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E. $(1/\sqrt{2})(\psi_1 - \psi_2)$

F. $Ax(L - x)$ (Choose this option if there is any value of A for which this is possible.)

Solution: A, B, D, E, and F. Option (F) doesn't look anything like our energy eigenstates, but it's continuous and normalizable so it is a valid wavefunction, and could in principle be written as a sum of eigenstates. Option (C), on the other hand, is not normalized.

Consider a double-well: $U(x) = 0$ for $0 < x < L$ and $2L < x < 3L$ and ∞ everywhere else. Which of the following are energy eigenstates of this system? For each one assume $\psi = 0$ in the regions where $U = \infty$, and assume A is chosen correctly to normalize the wavefunction. (Choose all that apply.)

A. $\psi(x) = A\sin(\pi x/L)$ for $0 < x < L$ and $\psi(x) = A\sin(\pi x/L)$ for $2L < x < 3L$.

B. $\psi(x) = A\sin(\pi x/L)$ for $0 < x < L$ and 0 for $2L < x < 3L$.

C. $\psi(x) = A\sin(\pi x/L)$ for $0 < x < L$ and $A\sin(2\pi x/L)$ for $2L < x < 3L$.

Consider a double-well: $U(x) = 0$ for $0 < x < L$ and $2L < x < 3L$ and ∞ everywhere else. Which of the following are energy eigenstates of this system? For each one assume $\psi = 0$ in the regions where $U = \infty$, and assume A is chosen correctly to normalize the wavefunction. (Choose all that apply.)

A. $\psi(x) = A\sin(\pi x/L)$ for $0 < x < L$ and $\psi(x) = A\sin(\pi x/L)$ for $2L < x < 3L$.

B. $\psi(x) = A\sin(\pi x/L)$ for $0 < x < L$ and 0 for $2L < x < 3L$.

C. $\psi(x) = A\sin(\pi x/L)$ for $0 < x < L$ and $A\sin(2\pi x/L)$ for $2L < x < 3L$.

Solution: A and B both solve Schrödinger's equation for $E = E_1$, so they are both energy eigenstates.

For C the first part solves Schrödinger's equation for $E = E_1$ and the second part solves it for $E = E_2$. That means there is no value of E for which this whole function is a solution to Schrödinger's equation, so this is not an energy eigenstate.

5.4 Other Bound States

Identify each of the following statements as true or false.

A. Bound states have discrete allowed energies.

B. For a bound state, the Schrödinger equation only has solutions for certain discrete values of E.

Identify each of the following statements as true or false.

A. Bound states have discrete allowed energies.

Solution: True

B. For a bound state, the Schrödinger equation only has solutions for certain discrete values of E.

Solution: False

Our first approach to Schrödinger's equation for the simple harmonic oscillator was to try $E = \hbar\omega$ and asking a computer to solve the resulting differential equation. Why did we try that particular combination, as opposed to (say) $E = m\omega^2$? (Choose one.)

A. Computer solutions reveal that $E = \hbar\omega$ works out better.

B. E must be a constant, and $m\omega^2$ isn't.

C. Setting $E = m\omega^2$ leads to solutions that cannot be normalized.

D. $E = \hbar\omega$ has the right units.

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C. Setting $E = m\omega^2$ leads to solutions that cannot be normalized.

D. $E = \hbar\omega$ has the right units.

Solution: D

Our solution to Schrödinger's equation for the finite square well looked like $e^{-x\sqrt{2m(U_0-E)}/\hbar}$ in the region $x \geq L$, but $e^{x\sqrt{2m(U_0-E)}/\hbar}$ in the region $x \leq 0$. Why wasn't there a negative-exponential term in $x \leq 0$? (Choose one.)

A. A negative-exponential term would not be normalizable in that region.

B. A negative-exponential term would not solve Schrödinger's equation in that region.

C. A negative-exponential term would not be able to meet the boundary conditions of both continuity and differentiability at $x = 0$.

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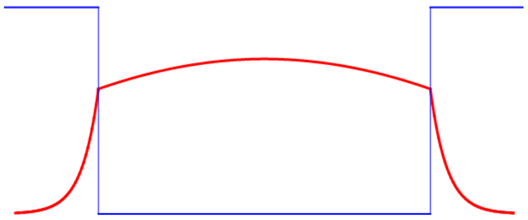
C. A negative-exponential term would not be able to meet the boundary conditions of both continuity and differentiability at $x = 0$.

Solution: A

The figure below shows a wavefunction that satisfies Schrödinger's equation inside and outside a finite square well. Why is it not an energy eigenstate of this system? (Choose one.)



A. It is not continuous.

B. It is not differentiable.

C. It is not normalizable.

D. It is a valid wavefunction, but it is not a state of definite energy.

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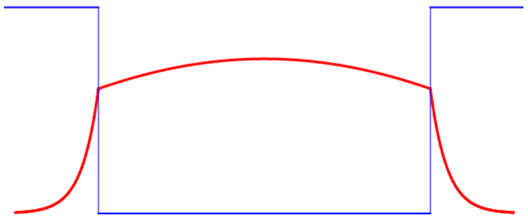


A. It is not continuous.

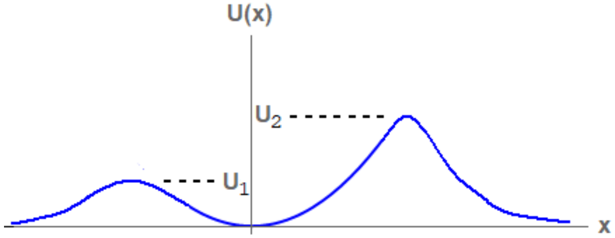
B. It is not differentiable.

C. It is not normalizable.

D. It is a valid wavefunction, but it is not a state of definite energy.

Solution: B

When does the potential field in the figure represent a bound quantum mechanical system? (Choose one.)



A. Never

B. Only when $E < U_2$ and the system is between the two peaks

C. Only when $E < U_1$ and the system is between the two peaks

D. Always

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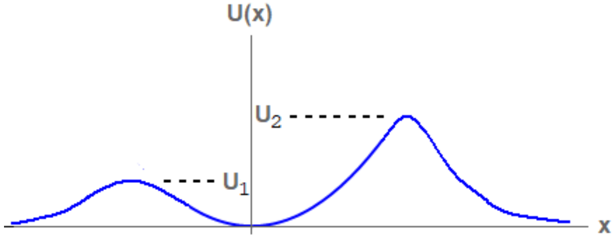

A. Never

B. Only when $E < U_2$ and the system is between the two peaks

C. Only when $E < U_1$ and the system is between the two peaks

D. Always

Solution: A

Can the ground state energy of a system ever be lower than the minimum value of its potential energy function?

Can the ground state energy of a system ever be lower than the minimum value of its potential energy function?

Solution: No, because kinetic energy cannot be negative.

A particle is in a finite square well, in an energy eigenstate with $E < U_0$. If you measure the particle's position, is it possible to find it in a region where $U = U_0$?

A particle is in a finite square well, in an energy eigenstate with $E < U_0$. If you measure the particle's position, is it possible to find it in a region where $U = U_0$?

Solution: Yes, it is. Our solution to the finite square well involved exponential functions that decay as $x \to \pm\infty$. So the probability gets smaller and smaller as you head deeper into $U = U_0$ territory, but it isn't zero.

We could instead have chosen $\psi(x) = 0$ for all the $U = U_0$ regions. That would still be a valid solution of Schrödinger's equation. But—assuming we don't set both A and B to zero, making $\psi = 0$ in the middle region as well (no particle at all)—we could not make the resulting function both continuous and differentiable at $x = 0$ and $x = L$. For the *infinite* square well we waived the requirement of differentiability, but here that requirement precludes such a solution.

5.5 Math Interlude: Complex Numbers

What is the imaginary part of the number $2 - 5i$? (Choose one.)

A. 2

B. -2

C. 5

D. -5

E. 29

F. $\sqrt{29}$

What is the imaginary part of the number $2 - 5i$? (Choose one.)

A. 2

B. -2

C. 5

D. -5

E. 29

F. $\sqrt{29}$

Solution: D

What is the modulus of the number $2 - 5i$? (Choose one.)

A. 2

B. -2

C. 5

D. -5

E. 29

F. $\sqrt{29}$

What is the modulus of the number $2-5i$? (Choose one.)

A. 2

B. -2

C. 5

D. -5

E. 29

F. $\sqrt{29}$

Solution: F

What is the complex conjugate of the number $2 - 5i$? (Choose one.)

A. $2 + 5i$

B. $2 - 5i$

C. $-2 + 5i$

D. $-2 - 5i$

E. $5 - 2i$

What is the complex conjugate of the number $2 - 5i$? (Choose one.)

A. $2 + 5i$

B. $2 - 5i$

C. $-2 + 5i$

D. $-2 - 5i$

E. $5 - 2i$

Solution: A

What is the phase of the number -29? (Choose one.)

A. 0

B. π

C. 29

D. -29

E. $\sqrt{29}$

What is the phase of the number -29? (Choose one.)

A. 0

B. π

C. 29

D. -29

E. $\sqrt{29}$

Solution: B

Which of the following expresses the number $e^{i\pi/3}$ in $a+bi$ form? (Choose one.)

A. $e + i\pi/3$

B. $e - i\pi/3$

C. $\cos(\pi/3) + i\sin(\pi/3)$

D. $\sin(\pi/3) + i\cos(\pi/3)$

Which of the following expresses the number $e^{i\pi/3}$ in $a+bi$ form? (Choose one.)

A. $e + i\pi/3$

B. $e - i\pi/3$

C. $\cos(\pi/3) + i\sin(\pi/3)$

D. $\sin(\pi/3) + i\cos(\pi/3)$

Solution: C

Which of the following is equal to e^{-ix}? (Choose one.)

A. $\cos x + i \sin x$

B. $\cos x - i \sin x$

C. $-\cos x + i \sin x$

D. $-\cos x - i \sin x$

E. None of the above

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A. $\cos x + i \sin x$

B. $\cos x - i \sin x$

C. $-\cos x + i \sin x$

D. $-\cos x - i \sin x$

E. None of the above

Solution: B

Which of the following represents all the points on the complex plane with ϕ equal to a constant? (Choose one.)

A. A line

B. A ray, or half-line

C. A circle

D. A wedge

E. None of the above.

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E. None of the above.

Solution: B

Does the function $z(t) = e^{it}$ represent …(Choose one.)

A. an increasing function

B. a decreasing function

C. a periodic function

D. a constant function

E. none of the above

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A. an increasing function

B. a decreasing function

C. a periodic function

D. a constant function

E. none of the above

Solution: C. Part of what makes this tricky is that the words "increasing" and "decreasing" don't apply to complex numbers. (More generally, the relationships "greater than" and "less than" apply only to real numbers.) The function e^{it} is periodic: it loops eternally around the unit circle. (So even if "increasing" was a valid option here, it could not possibly always apply to a periodic function!)

Which of the following represents all the points on the complex plane of the form $z = 5e^{i\alpha}$ where α can be any real number? (Choose one.)

A. A line

B. A half-line

C. A circle

D. A wedge

E. None of the above.

Which of the following represents all the points on the complex plane of the form $z = 5e^{i\alpha}$ where α can be any real number? (Choose one.)

A. A line

B. A half-line

C. A circle

D. A wedge

E. None of the above.

Solution: C

Which of the following represents all the points on the complex plane of the form $z = te^{it}$ where t can be any real number? (Choose one.)

A. A line

B. A half-line

C. A circle

D. A wedge

E. None of the above.

Which of the following represents all the points on the complex plane of the form $z = te^{it}$ where t can be any real number? (Choose one.)

A. A line

B. A half-line

C. A circle

D. A wedge

E. None of the above.

Solution: E: it's a spiral. Many students will rush to rewrite any such function in $a + bi$ form (sines and cosines), but you're ahead of the game if you can also work in the form $|z|e^{i\phi}$. In this case, both the phase ϕ and the modulus $|z|$ start at zero and increase over time. So you start at the origin and move around in a circle counter-clockwise, steadily increasing your distance to the origin.

Let $z_1(t) = e^{3it}$ and $z_2(t) = e^{(3i+2)t}$. Which of the following is true of the moduli $|z_1(t)|$ and $|z_2(t)|$? (Choose one.)

A. $|z_1(t)| > |z_2(t)|$

B. $|z_1(t)| < |z_2(t)|$

C. $|z_1(t)| = |z_2(t)|$

D. It depends on the time t.

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D. It depends on the time t.

Solution: D. The way to see this is to rewrite z_2 as $e^{2t}e^{3it}$. Then you can see that it has the modulus of z_1 (which is just 1) multiplied by e^{2t}, which is less than 1 for $t < 0$ and greater than 1 for $t > 0$.

If z is a complex number and you multiply it by e^{2i} does that …(Choose one.)

A. increase the modulus

B. decrease the modulus

C. not change the modulus

D. It depends on the original number z.

If z is a complex number and you multiply it by e^{2i} does that …(Choose one.)

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Solution: C

What is $\ln i$? Choose one of the following answers. *Hint:* Think about what a natural log means!

A. $-\pi$

B. i

C. $i\pi/2$

D. $i\pi$

E. $i \ln \pi$

F. Undefined

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D. $i\pi$

E. $i\ln\pi$

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Solution: C. You need a number z such that $e^z = i$. Since $e^{i\alpha}$ is a number with modulus 1 and phase α, and i has modulus 1 and phase $\pi/2$ (graph it on the complex plane and you can see this), $z = i\pi/2$.

5.6 Time Evolution of a Wavefunction

A particle is in an energy eigenstate ψ_1 with energy E_1. Which of the following will remain constant over time? (Choose all that apply.)

A. The probability of finding the particle between two points x_1 and x_2.

B. The probability of finding the particle with energy E_1.

C. The particle's wavefunction $\Psi(x, t)$.

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Solution: A, B

A particle is in a superposition of three energy eigenstates: $\psi(x) = A\psi_1(x) + B\psi_2(x) + C\psi_3(x)$. Which of the following will remain constant over time? (Choose all that apply.)

A. The probability of finding the particle between two points x_1 and x_2.

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B. The probability of finding the particle with energy E_1.

C. The particle's wavefunction $\Psi(x, t)$.

Solution: B

A particle is in a state that is neither a single energy eigenstate nor a superposition of energy eigenstates. Which of the following will remain constant over time? (Choose all that apply.)

A. The probability of finding the particle between two points x_1 and x_2.

B. The probability of finding the particle with energy E_1.

C. The particle's wavefunction $\Psi(x, t)$.

D. This question is invalid because such a state cannot happen.

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B. The probability of finding the particle with energy E_1.

C. The particle's wavefunction $\Psi(x, t)$.

D. This question is invalid because such a state cannot happen.

Solution: D

Suppose a particle in an infinite square well (from $x = 0$ to $x = L$) is not in an energy eigenstate. Answer each of the questions below with "Yes," "No," or "Not enough information."

1. Will $\langle x \rangle$, the expectation value of its position, change over time?
2. If you average the value of $\langle x \rangle$ over a long time, will it on average be $L/2$?
3. If you wait a long time will $\langle x \rangle$ eventually settle down to $L/2$ and stay there?

Suppose a particle in an infinite square well (from $x = 0$ to $x = L$) is not in an energy eigenstate. Answer each of the questions below with "Yes," "No," or "Not enough information."

1. Will $\langle x \rangle$, the expectation value of its position, change over time?

   Solution: Yes. The position probabilities change, so the expectation value changes. As we saw in the diagrams, it tends to oscillate from left to right and back again.

2. If you average the value of $\langle x \rangle$ over a long time, will it on average be $L/2$?

   Solution: Yes. Without doing any calculations, we can conclude that by symmetry.

3. If you wait a long time will $\langle x \rangle$ eventually settle down to $L/2$ and stay there?

   Solution: No, it will continue to oscillate.

Suppose you start two particles in the state $\psi(x) = Ae^{-k(x-x_0)^2}$ (a bump centered on $x = x_0$). One of the particles has zero potential energy everywhere (no force) and the other has a simple harmonic oscillator potential energy function. A short while later will the two particles' wavefunctions be the same or different?

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Solution: Different. The way you time-evolve the wavefunction begins by breaking it down into its eigenstates, and then multiplying each eigenstate by $e^{-iE_n t/\hbar}$ for its eigenvalue. These two systems have different potential energy functions, therefore different eigenstates and eigenvalues, therefore different time evolution.

We therefore arrive at the hopefully-not-startling conclusion that the way an object moves depends on the forces acting on it!

A particle in a simple harmonic oscillator is known at time $t = 0$ to have one of two energy levels: either E_1 (with probability P_1) or E_2 (with probability P_2). Which of the following best describes this particle a few seconds later?

A. It still definitely has one of those two energy levels, with those two probabilities.

B. It still definitely has one of those two energy levels, but the probabilities are different.

C. It might have energy levels other than those two.

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B. It still definitely has one of those two energy levels, but the probabilities are different.

C. It might have energy levels other than those two.

Solution: A

If two different energy eigenstates of a system have the same energy eigenvalue as each other, they are said to be “degenerate.” If the system of a state is currently the superposition of two degenerate eigenstates, will its position probabilities change over time?

If two different energy eigenstates of a system have the same energy eigenvalue as each other, they are said to be "degenerate." If the system of a state is currently the superposition of two degenerate eigenstates, will its position probabilities change over time?

Solution: No. Both terms will get multplied by the same exponential $e^{-iEt/\hbar}$, so the modulus squared $|\Psi|^2$ is constant. You could also get this result from time-energy uncertainty. Since the energy is exactly known, nothing measurable changes over time.

If we tell you the wavefunction of a particle in a finite square well, would you be able in principle to write it as a linear combination of the energy eigenstates of a simple harmonic oscillator? (Why you would want to is a question even we don't have an answer for.)

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Solution: Yes. That wavefunction for the finite square well could also be a wavefunction in a simple harmonic oscillator. (How do we know it could? Because it's continuous, differentiable, and normalized. End of story.) So it could be expressed as a linear combination of simple harmonic oscillator energy eigenstates.